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TIME SERIES IN M DIMENSIONS: AUTOREGRESSIVE MODELS.(U)

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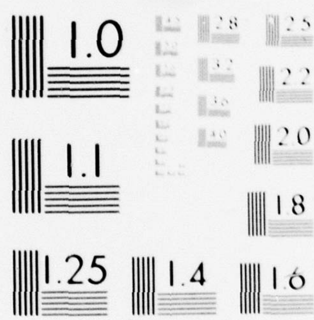
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TIME SERIES IN M-DIMENSIONS
AUTOREGRESSIVE MODELS

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TIME SERIES IN M DIMENSIONS

AUTOREGRESSIVE MODELS

by

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ABSTRACT

Spatially dependent autoregressive models in m dimensions are defined. The conditions for stationarity and invertibility are determined. The autocorrelation function and Yule-Walker equations are obtained for the general case, and as particular cases $z(t) = f(x_1, x_2; t)$, for $-\infty < t \leq t_0$, for special discrete values x_{1i}, x_{2i}, and t_i, for various grids in (x_1, x_2) plane and for orders 1 and 2 in time. The spectra are obtained for these particular cases, and some results for the partial autocorrelation function. All results are new. The notation, definitions and assumptions are those given by Voss et al (1977). We assume stationarity of $z_{x,t}$ over time t, where $x = (x_1, x_2, \ldots, x_m)$ an m dimensional vector. We assume the covariance structure as given by Hannan (1970), with $\sigma_z^2 > 0$, and all covariances existing. Non-stationary models will be considered in later papers.

1. Definitions, Model, Assumptions, Conditions.

The notation, definitions and assumptions are those given by Voss et al (1977). We assume stationarity of $z_{x,t}$ over time t, where $x = (x_1, x_2, \ldots, x_m)$ an m dimensional vector. We assume the covariance structure as given by Hannan (1970), with $\sigma_z^2 > 0$, and all covariances existing. Non-stationary models will be considered in later papers.

The autoregressive model in m dimensions is defined as

$$(1.1) \qquad \tilde{z}_{x,t} = \sum_{n=-\infty}^{\infty} \sum_{k=1}^{\infty} \varphi_{n,k} \tilde{z}_{x+n,\, t-k} + a_{x,t} .$$

In (1.1) $x_1, x_2, \ldots, x_m$ are m space variables, and t is the time variable. The a's are independent shocks, i.e., independent random variables with zero mean and variance of $\{a_{x,t}\} = \sigma_a^2$; $a_{x,t}$ is independent of $\tilde{z}_{x,\, t-k}$ except when k = 0. The autocorrelation function of the white noise process $a_{x,t}$ is given by $\rho_{0,0,\ldots,0,k} = \begin{matrix} 1 & k=0 \\ 0 & k \neq 0 \end{matrix}$ for $-\infty < t < \infty$. We assume that $\tilde{z}_{x,t}$ is a weakly stationary process. The autoregressive process can be thought of as the output $\tilde{z}_{x,t}$ from a linear filter with transfer function $\Phi^{-1}(B_x, B_t)$, when the input is white noise $a_{x,t}$. In this paper we are interested in some special cases of (1.1) in which only a finite number of coefficients are non-zero, i.e.,

$$(1.2) \qquad \tilde{z}_{x,t} = \sum_{n=-p}^{q} \sum_{k=1}^{r} \varphi_{n,k} \tilde{z}_{x+n,\, t-k} + a_{x,t} ,$$

where $p = (p_1, p_2, \ldots, p_m)$, and $q = (q_1, q_2, \ldots, q_m)$. In this case $\Phi(B_x, B_t)$ is finite. Therefore for invertibility, no restrictions are needed on the parameters $\varphi_{n,k}$. The autocovariance function of the autoregressive process, AR, is obtained by multiplying throughout by $\tilde{z}_{x-\ell,t-k}$ and then by taking expectations. We can obtain autocovariances via the autocovariance generating function $\Gamma(B_x, B_t) = \sum_{c=-\infty}^{\infty} \sum_{d=-\infty}^{\infty} \gamma_{cd} B_x^c B_t^d$, where γ_{cd} is the coefficient of both $B_x^c\, B_t^d$ and $B_x^{-c}\, B_t^{-d}$, and γ_{00} is the variance of the process. It can be shown that

(1.3) $\Gamma(B_x,B_t) = \sigma_a^2\, \Psi(B_x,B_t)\, \Psi(F_x F_t)$,

where $\Psi(B_x,B_t) = \Phi^{-1}(B_x,B_t)$ and $\Psi(F_x,F_t) = \Phi^{-1}(F_x,F_t)$.

Spectrum. If we substitute $B_t = e^{-i2\pi f}$ and $B_{x_j} = e^{-i2\pi g_j}$ for $1 \le j \le m$ and $i = \sqrt{-1}$ in (1.3), we obtain the power spectrum. Thus spectrum of AR process is

(1.4) $p(f,g) = 2\sigma_a^2 \left| \phi(e^{-i2\pi g}, e^{-i2\pi f}) \right|^{-2}$, $0 \le f \le \frac{1}{2}$, $0 \le g_j \le \frac{1}{2}$, $g = (g_1, g_2, \ldots, g_m)$, $\phi(e^{-i2\pi g}, e^{-i2\pi f}) = \phi(e^{-i2\pi g_1}, e^{-i2\pi g_2}, \ldots, e^{-i2\pi g_m}, e^{-i2\pi f})$.

Stationarity. The stationarity conditions for AR process are generalizations of Box and Jenkins, i.e., for stationarity $\phi^{-1}(B_x,B_t)$ must converge for some sets of values of B_x, B_t. These conditions are obtained for various models in the next two sections.

In section 2 we consider the case of m = 1, and in section 3 we consider the case of m = 2. In each of these sections, various models are discussed discussed in detail. All zero dimensional results of Box and Jenkins (1970) are special cases of the more general models in this paper.

2. Autoregressive Models in m = 1 dimensions.

For m = 1, (1.2) can be written as (writing x for x_1 and n for n_1)

(2.1) $\tilde{z}_{x,t} = \sum_{n=-p_1}^{q_1} \sum_{k=1}^{r} \varphi_{n,k}\, \tilde{z}_{x+n,t-k} + a_{x,t}$.

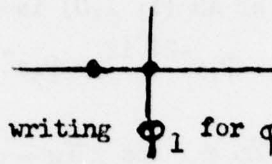
The process defined by (2.1) is denoted by AR (r; p_1, q_1) , and can be written as,

(2.2) $\phi(B_x,B_t)\, \tilde{z}_{x,t} = a_{x,t}$, where

(2.3) $\phi(B_x,B_t) = 1 - \sum_{n=-p_1}^{q_1} \sum_{k=1}^{r} \varphi_{n,k}\, B_x^n\, B_t^k$.

The autocorrelation function, Yule-Walker equations and stationarity conditions are obtained for various special cases of AR $(r;\ p_1,\ q_1)$. Since the series $\Phi(B_x, B_t)$ in (2.3) is finite, no restrictions are required to ensure invertibility.

2.1 The Model AR (1; 1,0)



From (2.1), AR (1; 1,0) writing φ_1 for φ_{01} and φ_2 for φ_{11}

(2.1.1) $\tilde{z}_{x,t} = \varphi_1 \tilde{z}_{x,t-1} + \varphi_2 \tilde{z}_{x-1,t-1} + a_{x,t}$, correspondingly

can be written as $\left(\Phi_1(B_x, B_t)\right) \tilde{z}_{x,t} = a_{x,t}$, where

(2.1.2) $\Phi_1(B_x, B_t) = 1 - (\varphi_1 + \varphi_2 B_x)\, B_t$.

Autocorrelation Function. Multiplying throughout in (2.1.1) by $\tilde{z}_{x-m,t-n}$, we obtain

(2.1.3) $\tilde{z}_{x-m,t-n}\tilde{z}_{x,t} = \varphi_1 \tilde{z}_{x-m,t-n}\tilde{z}_{x,t-1} + \varphi_2 \tilde{z}_{x-m,t-n}\tilde{z}_{x-1,t-1} + \tilde{z}_{x-m,t-n}\, a_{x,t}$.

On taking expected values in (2.1.3), we obtain

(2.1.4) $\gamma_{m,n} = \varphi_1 \gamma_{m,n-1} + \varphi_2 \gamma_{m-1,n-1}$ for

$m = 0,\ n \geq 1;\ m > 1,\ n = 0;$ and $m \geq 1,\ n \geq 1$.

(2.1.5) Also $\gamma_{00} = \varphi_1 \gamma_{01} + \varphi_2 \gamma_{11} + \sigma_a^2$, since $\gamma_{0-1} = \gamma_{01}$ and $\gamma_{-1-1} = \gamma_{11}$.

On dividing by $\gamma_{00} = \sigma_z^2$ in (2.1.4) and (2.1.5),

(2.1.6) $\rho_{m,n} = \varphi_1 \rho_{m,n-1} + \varphi_2 \rho_{m-1,n-1}$, either $m > 0$ or $n > 0$, but not $m = 1,\ n = 0$; and

(2.1.7) $1 = \varphi_1 \rho_{01} + \varphi_2 \rho_{11} + \sigma_a^2 / \sigma_z^2$.

From (2.1.4) and (2.1.6), we see that both autocorrelation and autocovariance functions satisfy the same form of difference equations. From (2.1.7), the variance σ_z^2 may be written as,

(2.1.8) $\sigma_z^2 = \sigma_a^2 \big/ (1 - \varphi_1 \rho_{01} - \varphi_2 \rho_{11})$.

From (2.1.2) $\Phi_1(B_x, B_t) = 1 - (\varphi_1 + \varphi_2 B_x)\, B_t$ and $\Psi(B_x, B_t) = \Phi_1^{-1}(B_x, B_t)$.

If we substitute $B_t = e^{-2\pi i f}$ and $B_x = e^{-2\pi i g}$, $i = \sqrt{-1}$ in the autocovariance generating function (2.1.8), we obtain the power spectrum. Thus the spectrum of AR (1; 1,0) is given by

$$(2.1.9)\quad p(f,g) = 2\sigma_a^2 \Big/ \left| 1 - \varphi_1 e^{-2\pi i f} - \varphi_2 e^{-2\pi i f} e^{-2\pi i g} \right|^2 =$$

$$2\sigma_a^2 \Big/ \left[1 + \varphi_1^2 + \varphi_2^2 + 2\varphi_1\varphi_2 \cos 2\pi g - 2\left(\varphi_1 \cos 2\pi f + \varphi_2 \cos 2\pi (f+g)\right)\right],$$

$$0 \le f \le \tfrac{1}{2},\ 0 \le g \le \tfrac{1}{2}.$$

Again when $\varphi_2 = 0$, we get the corresponding result for zero dimensions.

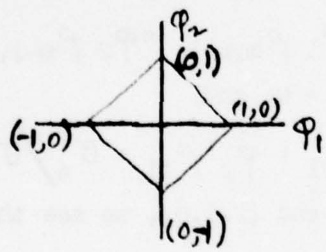
Stationarity. For stationarity, the autocovariances and autocorrelations must satisfy a set of conditions to ensure stationarity. The conditions can be combined in a single condition that the generating function $\psi(B_x, B_t)$ must converge for $|B_t| \le 1$ and $|B_x| \le 1$. See Theorem (2.1.12) below. Now

$$(2.1.10)\quad \psi(B_x, B_t) = \left(1 - (\varphi_1 + \varphi_2 B_x)\, B_t\right)^{-1} = \sum_{d=0}^{\infty} (\varphi_1 + \varphi_2 B_x)^d\, B_t^d\,.$$

From (2.1.10) we see that for stationarity $|\varphi_1 + \varphi_2 B_x| < 1$ and $|B_x| \le 1$. Taking $B_x = 1$ and $B_x = -1$, the parameters of AR (1; 1,0) must satisfy $|\varphi_1 + \varphi_2| < 1$, and $|\varphi_1 - \varphi_2| < 1$, to ensure stationarity. The two conditions can be combined into a single condition.

$$(2.1.11)\quad |\varphi_1| + |\varphi_2| < 1.$$

This region is shown in Fig. 1



We briefly give a justification for $|B_x| \le 1$. Operationally the condition for stationarity may be written as

$$\tilde{z}_{x,t} = \left(1 - (\varphi_1 + \varphi_2 B_x)\, B_t\right)^{-1} a_{x,t}$$

$$= \left\{ 1 + (\varphi_1 + \varphi_2 B_x)\, B_t + (\varphi_1 + \varphi_2 B_x)^2 B_t^2 + \ldots \right\} a_{x,t}$$

$$= a_{x,t} + (\varphi_1+\varphi_2 B_x)\, a_{x,t-1} + (\varphi_1+\varphi_2 B_x)^2\, a_{x,t-2} + \cdots$$
$$= a_{x,t} + \varphi_1 a_{x,t-1} + \varphi_2 a_{x-1,t-1} + \varphi_1^2 a_{x,t-2} + 2\varphi_1\varphi_2 a_{x-1,t-2} + \varphi_2^2 a_{x-2,t-2} + \cdots$$

This series does not involve B_t. Hence it is zero dimensional in x. So, $|B_x| \le 1$, as must be the case for stationarity for any B operator since we assume stationarity in $\{x, t\}$, x a vector.

(2.1.12) Generalization of the theorem in zero dimensions.

The roots of the characteristic equation must lie outside the unit circle in each variable, B_i when all other B's are set to one. The condition must also hold, as stationarity in m dimensions, m + 1 variables, requires stationarity in every direction in the m + 1 variables. The same condition holds in case of invertibility for the MA models.

Autoregressive parameters in terms of autocorrelations Yule-Walker equations.

From (2.1.6), we obtain $(\rho_{-10} = \rho_{10})$

(2.1.13) $\rho_{01} = \varphi_1 + \varphi_2\,\rho_{10}$

$\rho_{11} = \varphi_1\,\rho_{10} + \varphi_2$ Hence

(2.1.14) $\varphi_1 = (\rho_{01} - \rho_{11}\rho_{10}) \big/ (1-\rho_{10}^2)$

$\varphi_2 = (\rho_{11} - \rho_{01}\,\rho_{10}) \big/ (1-\rho_{10}^2)$.

Again from (2.1.6) we obtain

(2.1.15) $\rho_{0,k} = \varphi_1\,\rho_{0,k-1} + \varphi_2\,\rho_{1,k-1}$, $k > 1$

(2.1.16 $\rho_{k,0} = \varphi_1\,\rho_{k,1} + \varphi_2\,\rho_{k-1,1}$, $k > 1$ and

(2.1.17) $\rho_{k,k} = \varphi_1\,\rho_{k,k-1} + \varphi_2\,\rho_{k-1,k-1}$, $k \ge 1$.

Formulas (2.1.7), (2.1.13), (2.1.14), (2.1.15) and (2.1.16) can be used recursively to compute various autocorrelations.

A special case of (2.1.14) occurs if $\rho_{01} = \rho_{11}$. Then from (2.1.14) $\varphi_1 = \varphi_2 = \rho_{01}(1 + \rho_{10})^{-1}$, provided $\rho_{10} \neq 1$. As an example, if $\rho_{01} = \rho_{11} = \rho_{10} = \frac{1}{2}$, then $\varphi_1 = \varphi_2 = 1/3$.

Another special case occurs if $\rho_{10} = 1$, then from (2.1.11) we have $\rho_{01} = \varphi_1 + \varphi_2$, $\rho_{11} = \varphi_1 + \varphi_2$ which is meaningless unless $\rho_{01} = \rho_{11}$, and of course $\varphi_1 + \varphi_2 = \rho_{01} = \rho_{11}$. Clearly this means $|\varphi_1 + \varphi_2| < 1$, one condition as already seen in (2.1.10) for invertibility. Such cases however are singular since we insist $1 - \rho_{10}^2 > 0$.

Further we note that φ_1 and φ_2 are determined by ρ_{01}, ρ_{10}, and ρ_{11}. Conversely if φ_1 and φ_2 are chosen in accordance with (2.1.13), then given any one of ρ_{01}, ρ_{10}, or ρ_{11}, the other two are determined.

(2.1.18) As an example we take $\varphi_1 = .2$, $\varphi_2 = -.6$,

$$\tilde{z}_{xt} = .2\,\tilde{z}_{x,t-1} - .6\,\tilde{z}_{x-1,t-1} + a_{xt}.$$

The set of values for the ρ's is determined by the equations (2.1.3), given φ_1, φ_2 and ρ_{10} we solve for ρ_{01} and ρ_{11}. We choose $\rho_{10} = .5$ and consequently $\rho_{01} = -.1$, $\rho_{11} = -.5$. The autocorrelation function is given by (2.1.6), (2.1.15), (2.1.16), and (2.1.17). In all cases $\rho_{m1} = -\rho_{m0}$, $m > 0$. For the special case $\varphi_1 = .2$, $\varphi_2 = -.6$, the autocorrelation function ρ_{mn} is given by the diagram where the x axis represents m and the y axis n.

n \ m	0	1	2	3	4	5
1	1	.5	.25	.125	.0625	.03125
2	-.1	-.5	-.25	-.125	-.0625	-.03125
3	.28	-.04	.25	.125	.0625	
4	.08	-.18	.074	-.125		
5	.131	-.083	.1204			
6	.076	-.095				
7	.072					

The analysis for the model AR (1; 0,1) is given by

(2.1.19) $\tilde{z}_{xt} = \varphi_1 \tilde{z}_{x,t-1} + \varphi_3 \tilde{z}_{x+1,t-1} + a_{xt}$.

It is similar to model AR(1; 1,0). The results are analogous if we replace the parameter φ_2 by φ_3 and the operator B_x by F_x.

2.2 The Model AR (2; 1,0). AR (2;1,0) is given by

(2.2.1) $\tilde{z}_{xt} = \varphi_{01} \tilde{z}_{x,t-1} + \varphi_{11} \tilde{z}_{x-1,t-1} + \varphi_{02} \tilde{z}_{x,t-2} + \varphi_{12} \tilde{z}_{x-1,t-2} + a_{xt}$, or

(2.2.2) $\Phi_2 (B_x, B_t) \tilde{z}_{xt} = a_{xt}$ where

(2.2.3) $\Phi_2 (B_x, B_t) = 1 - (\varphi_{01} + \varphi_{11} B_x + \varphi_{02} B_t + \varphi_{12} B_x B_t) B_t$.

Conditions satisfied by the autocorrelations. The correlation matrices for the Yule-Walker equations in t and x are,

(2.2.4)

$$\sigma_z^2 P_x = \sigma_z^2 \begin{bmatrix} 1 & \rho_{10} \\ \rho_{10} & 1 \end{bmatrix} \text{ and } \sigma_z^2 P_t = \sigma_z^2 \begin{bmatrix} 1 & \rho_{01} & \rho_{02} \\ \rho_{01} & 1 & \rho_{01} \\ \rho_{02} & \rho_{01} & 1 \end{bmatrix} .$$

From (2.2.4), positive definiteness implies $1 - \rho_{ij}^2 > 0$,

(2.2.5)
$$\begin{vmatrix} 1 & \rho_{01} & \rho_{02} \\ \rho_{01} & 1 & \rho_{01} \\ \rho_{02} & \rho_{01} & 1 \end{vmatrix} > 0, \text{ and } \begin{vmatrix} 1 & \rho_{01} & \rho_{11} & \rho_{02} & \rho_{12} \\ \rho_{01} & 1 & \rho_{10} & \rho_{01} & \rho_{11} \\ \rho_{11} & \rho_{10} & 1 & \rho_{11} & \rho_{01} \\ \rho_{02} & \rho_{01} & \rho_{11} & 1 & \rho_{10} \\ \rho_{12} & \rho_{11} & \rho_{01} & \rho_{10} & 1 \end{vmatrix} > 0.$$

Autocorrelation function. Multiplying (2.2.1) by $\tilde{z}_{x-m,t-n}$, we get

$$\tilde{z}_{x-m,t-n} \tilde{z}_{xt} = \varphi_{01} \tilde{z}_{x-m,t-n} \tilde{z}_{x,t-1} + \varphi_{11} \tilde{z}_{x-m,t-n} \tilde{z}_{x-1,t-1} + \varphi_{02} \tilde{z}_{x,t-2} \tilde{z}_{x-m,t-n}$$
$$+ \varphi_{12} \tilde{z}_{x-m,t-n} \tilde{z}_{x-1,t-2} + \tilde{z}_{x-m,t-n} a_{xt}.$$

Taking expected values on both sides, we get

(2.2.6) $\gamma_{mn} = \varphi_{01} \gamma_{m,n-1} + \varphi_{11} \gamma_{m-1,n-1} + \varphi_{02} \gamma_{m,n-2} + \varphi_{12} \gamma_{m-1,n-2}$

for $m = 0$, $n \geq 1$; $m > 1$, $n = 0$; $n \geq 1$, $m \geq 1$.

Also

(2.2.7) $\gamma_{00} = \varphi_{01}\gamma_{01} + \varphi_{11}\gamma_{11} + \varphi_{02}\gamma_{02} + \varphi_{12}\gamma_{12} + \sigma_a^2$.

Dividing by $\gamma_{00} = \sigma_z^2$ in (2.2.6) and (2.2.7)

(2.2.8) $\rho_{mn} = \varphi_{01}\rho_{m,n-1} + \varphi_{11}\rho_{m-1,n-1} + \varphi_{02}\rho_{m,n-2} + \varphi_{12}\rho_{m-1,n-2}$

for $m = 0, n \geq 1; m>1, n = 0; m \geq 1, n \geq 1$. And

(2.2.9) $1 = \varphi_{01}\rho_{01} + \varphi_{11}\rho_{11} + \varphi_{02}\rho_{02} + \varphi_{12}\rho_{12} + \sigma_a^2/\sigma_z^2$.

The variance σ_z^2 from (2.2.9) is given by,

(2.2.10) $\sigma_z^2 = \sigma_a^2 / (1 - \varphi_{01}\rho_{01} - \varphi_{11}\rho_{11} - \varphi_{02}\rho_{02} - \varphi_{12}\rho_{12})$.

Spectrum. Let $\psi(B_x,B_t) = \Phi_2^{-1}(B_x,B_t)$, and from (2.2.3)

$\Phi_2(B_x,B_t) = 1 - (\varphi_{01} + \varphi_{11}B_x + \varphi_{02}B_t + \varphi_{12}B_xB_t)\, B_t$.

Substituting $B_t = e^{-2i\pi f}$ and $B_x = e^{-2i\pi g}$ in the autocovariance generating function $\Gamma(B_x,B_t)$ given in (1.3), we get the power spectrum. Thus the spectrum of AR (2; 1,0) is given by

(2.2.11)

$$p(f,g) = 2\sigma_a^2 \Big/ \left|1 - \varphi_{01}e^{-2i\pi f} - \varphi_{11}e^{-2i\pi f}e^{-2i\pi g} - \varphi_{02}e^{-4i\pi f} - \varphi_{12}e^{-4i\pi f}e^{-2i\pi g}\right|^2$$

$0 \leq f \leq \frac{1}{2},\quad 0 \leq g \leq \frac{1}{2}$.

Stationarity. As in section 2.1, for stationarity $\psi(B_x,B_t) = \Phi^{-1}(B_x,B_t)$

$= \left[1 - (\varphi_{01} + \varphi_{11}B_x + \varphi_{02}B_t + \varphi_{12}B_xB_t)\ B_t\right]^{-1}$

$= \sum_{d=0}^{\infty} \left[(\varphi_{01} + \varphi_{02}B_t) + (\varphi_{11}B_x + \varphi_{12}B_xB_t)\right]^d B_t^d$

must converge for $|B_t| \leq 1$ and $|B_x| \leq 1$.

This implies that the roots of $1 - \left((\varphi_{01} + \varphi_{11}B_x)B_t + (\varphi_{02} + \varphi_{12}B_x)B_t^2\right) = 0$ must lie outside the region $|B_t| \leq 1$ and $|B_x| \leq 1$. The conditions are

(2.2.12)
$$\begin{aligned}
\varphi_{02} + \varphi_{12} + \varphi_{01} + \varphi_{11} &< 1\\
\varphi_{02} + \varphi_{12} - \varphi_{01} - \varphi_{11} &< 1\\
\varphi_{02} - \varphi_{12} + \varphi_{01} - \varphi_{11} &< 1\\
\varphi_{02} - \varphi_{12} - \varphi_{01} + \varphi_{11} &< 1 \quad \text{and}\\
|\varphi_{02}| + |\varphi_{12}| &< 1.
\end{aligned}$$

Yule-Walker Equations.

Multiplying in (2.2.1) by $\tilde{z}_{x,t-1}$, $\tilde{z}_{x-1,t-1}$, $\tilde{z}_{x,t-2}$, and $\tilde{z}_{x-1,t-2}$ respectively and taking expected values on both sides, we get,

(2.2.14) $\rho_{01} = \varphi_{01} + \varphi_{11}\rho_{10} + \varphi_{02}\rho_{01} + \varphi_{12}\rho_{11}$,

(2.2.15) $\rho_{11} = \varphi_{01}\rho_{10} + \varphi_{11} + \varphi_{02}\rho_{11} + \varphi_{12}\rho_{01}$,

(2.2.16) $\rho_{02} = \varphi_{01}\rho_{01} + \varphi_{11}\rho_{11} + \varphi_{02} + \varphi_{12}\rho_{10}$, and

(2.2.17) $\rho_{12} = \varphi_{01}\rho_{11} + \varphi_{11}\rho_{01} + \varphi_{02}\rho_{10} + \varphi_{12}$.

These equations can be solved and parameters φ_{01}, φ_{11}, φ_{02}, φ_{12} can be expressed in terms of autocorrelations. Also from (2.2.8) we obtain

(2.2.18) $\rho_{0k} = \varphi_{01}\rho_{0,k-1} + \varphi_{11}\rho_{1,k-1} + \varphi_{02}\rho_{0,k-2} + \varphi_{12}\rho_{1,k-2}$,

(2.2.19) $\rho_{k0} = \varphi_{01}\rho_{k1} + \varphi_{11}\rho_{k-1,1} + \varphi_{02}\rho_{k2} + \varphi_{12}\rho_{k-1,1}$, and

(2.2.20) $\rho_{kk} = \varphi_{01}\rho_{k,k-1} + \varphi_{11}\rho_{k-1,k-1} + \varphi_{02}\rho_{k,k-2} + \varphi_{12}\rho_{k-1,k-2}$,

where $k > 1$. Formulas (2.2.18), (2.2.19), (2.2.20), and (2.2.8) can be used recursively to compute various autocorrelations.

2.3 Partial autocorrelation function for AR (1; 1,0). The partial autocorrelation is a device which helps us decide which order autoregressive process to fit. With the models given in sections 2.1 and 2.2, we prove the following result.

Theorem. If the true model is AR (1; 1,0) , then $\varphi_{02} = 0$ and $\varphi_{12} = 0$, assuming $\rho_{01} \neq \rho_{11}$.

Proof. From (2.2.14) and (2.2.15) ,

$$\rho_{01}^2 - \rho_{11}^2 = \varphi_{01}(\rho_{01} - \rho_{10}\rho_{11}) + \varphi_{11}(\rho_{10}\rho_{01} - \rho_{11}) + \varphi_{02}(\rho_{01}^2 - \rho_{11}^2)$$

Substituting the values of $\varphi_{01} = \varphi_1$ and $\varphi_{11} = \varphi_2$ from (2.1.12) and after simplification, we get $\varphi_{02}(\rho_{01}^2 - \rho_{11}^2)(1 - \rho_{10}^2) = 0$.

But $1 - \rho_{10}^2 \neq$ by (2.2.5) and $\rho_{01}^2 - \rho_{11}^2 \neq 0$ by assumption, therefore $\varphi_{02} = 0$. Similarly it can be shown that $\varphi_{12} = 0$, which completes the proof. It should be noticed that all other parameters like φ_{0k}, φ_{kk}, and φ_{1k} for $k \geq 2$ are zero.

2.4 The Model AR (1; 1,1)

The model AR(1; 1,1) is given by

$\tilde{z}_{xt} = \varphi_{01}\tilde{z}_{x,t-1} + \varphi_{-11}\tilde{z}_{x-1,t-1} + \varphi_{11}\tilde{z}_{x+1,t-1} + a_{xt}$, which reduces immediately to

(2.4.1) $\tilde{z}_{xt} = \varphi_{01}\tilde{z}_{x,t-1} + \varphi_{11}(\tilde{z}_{x-1,t-1} + \tilde{z}_{x+1,t-1}) + a_{xt}$ since $\varphi_{-11} = \varphi_{11}$ as $\rho_{-11} = \rho_{11}$.

(2.4.2) Hence $\Phi(B_x,F_x,B_t) = 1 - \left(\varphi_{01} + \varphi_{11}(B_x+F_x)\right) B_t$.

The autocorrelation function is found by multiplying (2.4.1) by $\tilde{z}_{x-m,t-n}$ to get,

(2.4.3) $\tilde{z}_{x-m,t-n}\tilde{z}_{xt} = \varphi_{01}\tilde{z}_{x-m,t-n}\tilde{z}_{x,t-1} + \varphi_{11}\tilde{z}_{x-m,t-n}(\tilde{z}_{x-1,t-1} + \tilde{z}_{x+1,t-1})$
$+ \tilde{z}_{x-m,t-n}\, a_{xt}$. We take expected values to get

(2.4.4) $\gamma_{m,n} = \varphi_{01}\gamma_{m,n-1} + \varphi_{11}(\gamma_{m-1,n-1} + \gamma_{m+1,n-1})$,

(2.4.5) $\gamma_{00} = \varphi_{01}\gamma_{01} + 2\varphi_{11}\gamma_{11} + \sigma_a^2$,

(2.4.6) $\rho_{m,n} = \varphi_{01}\rho_{m,n-1} + \varphi_{11}(\rho_{m-1,n-1} + \rho_{m+1,n-1})$, $m = 0,\ n \geq 1$;
$m > 1,\ n = 0$; $m \geq 1,\ n \geq 1$. From (2.4.5) if we divide by σ_z^2,

(2.4.7) $\sigma_z^2 = \sigma_a^2 (1 - \varphi_{01}\rho_{01} - 2\varphi_{11}\rho_{11})^{-1}$.

The power spectrum is found as usual by substitution of $B_t = e^{-2i\pi f}$, $B_x = e^{-2i\pi g}$, $F_x = e^{2i\pi g}$, $i = \sqrt{-1}$ in the autocovariance generating function $\Gamma(B_x,F_x,B_t) = \sigma_a^2 \Psi(B_x,F_x,B_t)\,\Psi(F_x,B_xF_t)$. Thus $p(f,g)$ is given by

(2.4.8) $p(f,g) = 2\sigma_a^2 \left| 1 - \varphi_{01}e^{-2i\pi f} - \varphi_{11}(e^{-2i\pi g} + e^{2i\pi g}) \right|^{-2}$,
$0 \leq f \leq \tfrac{1}{2}$, $0 \leq g \leq \tfrac{1}{2}$.

The conditions for stationarity are found by setting $B_x = \pm 1$, $F_x = \pm 1$,

and $B_t = \pm 1$ using the same arguments as before. Now $\psi(B_x, F_x, B_t) = \Phi^{-1}(B_x, F_x, B_t)$
$= \left\{1 - \left(\varphi_{01} + \varphi_{11}(B_x + F_x)\, B_t\right)\right\}^{-1} = \sum_0^\infty \left(\varphi_{01} + \varphi_{11}\,(B_x + F_x)\right)^d\, B_t^d$.
Hence the restrictions on the parameters are: $|\varphi_{01} + 2\varphi_{11}| < 1$, $|\varphi_{01} - 2\varphi_{11}| < 1$,
(2.4.9) or $|\varphi_{01}| + 2|\varphi_{11}| < 1$, similar to the results given in (2.1).

The Yule-Walker equations are found from (2.4.6) using $m=0$, $n=1$, and $m = n = 1$:

(2.4.10) $\rho_{01} = \varphi_{01} + 2\varphi_{11}\rho_{10}$, $\rho_{11} = \varphi_{01}\rho_{10} + \varphi_{11}(1 + \rho_{20})$. Consequently

(2.4.11)
$$\varphi_{01} = \left(\rho_{01}(1+\rho_{20}) - 2\rho_{10}\rho_{11}\right) / \left(1 + \rho_{20} - 2\rho_{10}^2\right)$$
$$\varphi_{11} = \left(\rho_{11} - \rho_{01}\rho_{10}\right) / \left(1 + \rho_{20} - \rho_{10}^2\right).$$

The recursion formulas for the autocorrelation function are :

(2.4.12)
$$\rho_{0k} = \varphi_{01}\rho_{0,k-1} + 2\varphi_{11}\rho_{1,k-1}, \quad \rho_{k0} = \varphi_{01}\rho_{k,1} + \varphi_{11}(\rho_{k-1,1} + \rho_{k+1,1}),$$
$$\rho_{kk} = \varphi_{01}\rho_{k,k-1} + \varphi_{11}(\rho_{k-1,k-1} + \rho_{k+1,k-1}), \quad k \geq 1.$$

Now $\varphi_{11} = 0$, if $\rho_{11} = \rho_{01}\rho_{10}$, and if $\varphi_{11} = 0$, $\rho_{11} = \rho_{01}\rho_{10}$ from (2.4.10). If however $1 + \rho_{20} - 2\rho_{10}^2 = 0$, then as a consequence $\rho_{11} = \rho_{01}\rho_{10}$, and then $\varphi_{11} = 0$. The condition $1 + \rho_{20} - 2\rho_{10}^2 > 0$ is satisfied when $m = 0$, or $\varphi_{11} = 0$.

3. Autoregressive Models in M = 2 Dimensions

For $m = 2$, (1.1) can be written as

(3.1)
$$\tilde{z}_{x_1,x_2,t} = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \sum_{k=1}^{\infty} \varphi_{n_1,n_2,k}\, \tilde{z}_{x_1+n_1,x_2+n_2,t-k} + a_{x_1,x_2,t}.$$

Consider a special case of (3.1) in which only a finite number of φ's are non-zero, i.e., $m = 2$, in (1.2) ,

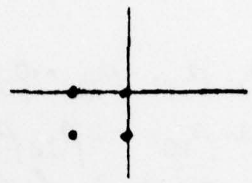
(3.2)
$$\tilde{z}_{x_1,x_2,t} = \sum_{n_1=-p_1}^{q_1} \sum_{n_2=-p_2}^{q_2} \sum_{k=1}^{r} \varphi_{n_1,n_2,k}\, \tilde{z}_{x_1+n_1,x_2+n_2,t-k} + a_{x_1,x_2,t}.$$

The autoregressive process defined by (3.2) is denoted as $AR(r;p_1,q_1;p_2,q_2)$, and can be written as $\Phi_3(B_{x_1}, B_{x_2}, B_t)\,\tilde{z}_{x_1,x_2,t} = a_{x_1,x_2,t}$, where

$$(3.3)\quad \Phi_3(B_{x_1},B_{x_2},B_t) = 1 - \sum_{n_1=-p_1}^{q_1} \sum_{n_2=-p_2}^{q_2} \sum_{k=1}^{r} \varphi_{n_1,n_2,k}\, B_{x_1}^{n_1} B_{x_2}^{n_2} B_t^{k} .$$

Autocorrelation function, stationarity conditions and Yule-Walker equations are obtained for various special cases of AR(r; p_1,q_1; p_2,q_2). Clearly various results obtained reduce to the corresponding results for m = 0 or 1.

3.1 The Model AR (1; 1,0; 1,0).



From (3.2), AR (1; 1,0; 1,0) is

$$(3.1.1)\quad \tilde{z}_{xyt} = \varphi_{001}\,\tilde{z}_{xy,t-1} + \varphi_{101}\tilde{z}_{x-1,y,t-1} + \varphi_{011}\tilde{z}_{x,y-1,t-1}$$

$+ \varphi_{111}\tilde{z}_{x-1,y-1,t-1} + a_{xyt}$, denoting x_1 by x and x_2 by y for convenience and also φ_{-101} by φ_{101}, φ_{0-11} by φ_{011}.

First we consider two interesting special cases of (3.1.1)

Special Case 1. Let $\varphi_{111} = 0$, i.e., we consider the model

$$(3.1.2)\quad \tilde{z}_{xyt} = \varphi_{001}\tilde{z}_{xy,t-1} + \varphi_{101}\tilde{z}_{x-1,y,t-1} + \varphi_{011}\tilde{z}_{x,y-1,t-1} + a_{xyt}$$

or

$$(3.1.3)\quad \Phi_{31}(B_x,B_y,B_t)\,\tilde{z}_{xyt} = a_{xyt}\text{ , where}$$

$$(3.1.4)\quad \Phi_{31}(B_x,B_y,B_t) = 1 - (\varphi_{001} + \varphi_{101}B_x + \varphi_{011}B_y)\,B_t .$$

Autocorrelation function. Multiplying in (3.1.2) by $\tilde{z}_{x-\ell,y-m,t-n}$ and taking the expected values, we obtain

$$(3.1.5)\quad \gamma_{\ell mn} = \varphi_{001}\gamma_{\ell,m,n-1} + \varphi_{101}\gamma_{\ell-1,m,n-1} + \varphi_{011}\gamma_{\ell,m-1,n-1}$$ and

at least one of ℓ,m,n greater than zero and others zero or positive,

$$(3.1.6)\quad \gamma_{000} = \varphi_{001}\gamma_{001} + \varphi_{101}\gamma_{101} + \varphi_{011}\gamma_{011} + \sigma_a^2 .$$

From (3.1.5) on dividing by $\gamma_{000} = \sigma_z^2$, we get the autocorrelation function

$$(3.1.7)\quad \rho_{\ell mn} = \varphi_{001}\rho_{\ell,m,n-1} + \varphi_{101}\rho_{\ell-1,m,n-1} + \varphi_{011}\rho_{\ell,m-1,n-1}$$, at

least one of ℓ, m, n > 0, and others zero or positive. From (3.1.6), the variance σ_z^2 is obtained on dividing by γ_{000} , as

$$(3.1.8)\quad \sigma_z^2 = \sigma_a^2 \Big/ \left(1 - \varphi_{001}\gamma_{001} - \varphi_{101}\gamma_{101} - \varphi_{011}\gamma_{011}\right).$$

Spectrum. Let $\Psi(B_x,B_y,B_t) = \Phi^{-1}_{31}(B_x,B_y,B_t)$, and from (3.1.4), $\Phi(B_x,B_y,B_t) = 1 - (\varphi_{001} + \varphi_{101}B_x + \varphi_{011}B_y)\,B_t$. Let $B_t = e^{-2i\pi f}$, $B_x = e^{-2i\pi g}$, and $B_y = e^{-2i\pi h}$ in the autocovariance generating function, $\Gamma = \sigma_a^2\,\Psi(B_x,B_y,B_t)\,\Psi(F_x,F_y,F_t)$, the spectrum of autoregressive model (3.1.2) is given by

(3.1.9) $$p(f,g,h) = 2\sigma_a^2 \Big/ \left|1 - (\varphi_{001} + \varphi_{101}e^{-2i\pi g} + \varphi_{011}e^{-2i\pi h})\,e^{-2i\pi f}\right|^2 ,$$
$$0 \le f \le \tfrac{1}{2},\quad 0 \le g \le \tfrac{1}{2},\quad 0 \le h \le \tfrac{1}{2}.$$

Stationarity. From stationarity $\psi_{31}(B_x,B_y,B_t)$ must converge for $|B_t| \le 1$, $|B_x| \le 1$, and $|B_y| \le 1$. From (3.1.4)

(3.1.10) $$\psi_{31}(B_x,B_y,B_t) = \left(1 - (\varphi_{001} + \varphi_{101}B_x + \varphi_{011}B_y)\;B_t\right)^{-1}$$
$$= \sum_{d=0}^{\infty} (\varphi_{001} + \varphi_{101}B_x + \varphi_{011})^d\; B_t^d .$$ As in (2.1.10), for stationarity

(3.1.11) $|\varphi_{001} + \varphi_{101}B_x + \varphi_{011}| < 1$ for $|B_x| \le 1$, $|B_y| \le 1$. Taking $B_x = 1, -1$ and $B_y = 1, -1$, the stationarity conditions are:

(3.1.12) $$|\varphi_{001} + \varphi_{101} + \varphi_{011}| < 1$$
$$|\varphi_{001} - \varphi_{101} + \varphi_{011}| < 1$$
$$|\varphi_{001} + \varphi_{101} - \varphi_{011}| < 1$$
$$|\varphi_{001} - \varphi_{101} - \varphi_{011}| < 1 .$$

Yule-Walker Equations. Multiplying throughout by $\tilde{z}_{x,y,t-1}$, $\tilde{z}_{x,y-1,t-1}$, and $\tilde{z}_{x-1,y,t-1}$ in (3.1.2) and taking the expected values, we get

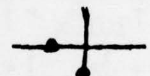
(3.1.13) $$\begin{aligned} \rho_{001} &= \varphi_{001} + \varphi_{101}\rho_{100} + \varphi_{011}\rho_{010} \\ \rho_{101} &= \varphi_{001}\rho_{100} + \varphi_{101} + \varphi_{011}\rho_{110} \\ \rho_{011} &= \varphi_{001}\rho_{010} + \varphi_{101}\rho_{110} + \varphi_{011} \end{aligned}$$. Let

(3.1.14) $$D = \begin{vmatrix} 1 & \rho_{100} & \rho_{010} \\ \rho_{100} & 1 & \rho_{110} \\ \rho_{010} & \rho_{110} & 1 \end{vmatrix} = 1 - \rho_{010}^2 - \rho_{100}^2 - \rho_{110}^2 + 2\rho_{010}\rho_{100}\rho_{010}$$

(3.1.15)
$$D^{-1}\begin{vmatrix} \rho_{001} & \rho_{100} & \rho_{010} \\ \rho_{101} & 1 & \rho_{110} \\ \rho_{011} & \rho_{110} & 1 \end{vmatrix} .$$

(3.1.16)
$$\varphi_{101} = D^{-1}\begin{vmatrix} 1 & \rho_{001} & \rho_{010} \\ \rho_{100} & \rho_{101} & \rho_{110} \\ \rho_{010} & \rho_{011} & 1 \end{vmatrix} .$$

(3.1.17)
$$\varphi_{011} = D^{-1}\begin{vmatrix} 1 & \rho_{100} & \rho_{001} \\ \rho_{100} & 1 & \rho_{101} \\ \rho_{010} & \rho_{110} & \rho_{011} \end{vmatrix} .$$

Special Case 2. Let $\varphi_{111} = 0$ and $\varphi_{001} = 0$. This reduces to a time series in m = 1 dimensions and the model is  .

(3.1.18) $\tilde{z}_{xyt} = \theta_1 \tilde{z}_{x-1,y,t-1} + \theta_2 \tilde{z}_{x,y-1,t-1} + a_{xyt}$, $\varphi_{101} = \theta_1$, $\varphi_{0-11} = \theta_2$ or

$$(1 - \theta_1 B_x B_t - \theta_2 B_y B_t)\, \tilde{z}_{xyt} = a_{xyt}.$$

Autocorrelation function. Multiplying throughout in (3.1.18) by $\tilde{z}_{x-\ell,y-m,t-n}$ and taking expected values, we obtain

(3.1.19) $\gamma_{\ell,m,n} = \theta_1 \gamma_{\ell-1,m,n-1} + \theta_2 \gamma_{\ell,m-1,n-1}$, at least one of $\ell,m,n > 0$, and

(3.1.20) $\gamma_{000} = \theta_1 \gamma_{101} + \theta_2 \gamma_{011} + \sigma_a^2$. Dividing by γ_{000} we get from (3.1.20)

(3.1.21) $\rho_{\ell,m,n} = \theta_1 \rho_{\ell-1,m,n-1} + \theta_2 \rho_{\ell,m-1,n-1}$, and

(3.1.22) $1 = \theta_1 \rho_{101} + \theta_2 \rho_{011} + \sigma_a^2 / \sigma_z^2$. From (3.1.22), the variance σ_z^2 is given by $\sigma_z^2 = \sigma_a^2 / (1 - \theta_1 \rho_{101} - \theta_2 \rho_{011})$.

Spectrum. From (3.1.18)

(3.1.23) $\phi(B_x, B_y, B_t) = 1 - (\theta_1 B_x + \theta_2 B_y) B_t$.

(3.1.24) Let $\psi(B_x,B_y,B_t) = \phi^{-1}(B_x,B_y,B_t)$. The autocovariance generating function is given by $\Gamma = \sigma_a^2 \psi(B_x,B_y,B_t)\psi(F_x,F_y,F_t)$. Let $B_t = e^{-2i\pi f}$, $B_x = e^{-2i\pi g}$, and $B_y = e^{2i\pi h}$. The spectrum of the AR process (3.1.18) is given by

(3.1.25) $p(f,g,h) = 2\sigma^2_a \Big/ \left| 1 - (\theta_1 e^{-2i\pi g} + \theta_2 e^{-2i\pi h})\, e^{-2i\pi f} \right|^2$;

$0 \le f \le \frac{1}{2}, \quad 0 \le g \le \frac{1}{2}, \quad 0 \le h \le \frac{1}{2}.$

Stationarity. Expanding (3.1.18) as in (2.1.10), the stationarity conditions are

$|\theta_1 B_x + \theta_2 B_y| < 1, \quad |B_x| \le 1$, and $|B_y| \le 1$, which gives

(3.1.26) $|\theta_1 + \theta_2| < 1 \quad , \quad |\theta_1 - \theta_2| < 1.$ These conditions are similar to (2.1.10).

Yule-Walker Equations. Multiplying by $\tilde{z}_{x-1,y,t-1}$ and $\tilde{z}_{x,y-1,t-1}$ and taking expected values on both sides, dividing by $\gamma_{000} = \sigma^2_z$, we get

(3.1.27) $\rho_{101} = \theta_1 + \theta_2 \rho_{110}$ and

(3.1.28) $\rho_{011} = \theta_1 \rho_{110} + \theta_2$. Solving, we get

(3.1.29) $\theta_1 = (\rho_{101} - \rho_{011}\rho_{110}) / (1 - \rho^2_{110})$

$\theta_2 = (\rho_{011} - \rho_{101}\rho_{110}) / (1 - \rho^2_{110})$.

This model and all *results obtained* are *similar* to the *m* = 1 dimensional model considered in section 2.1

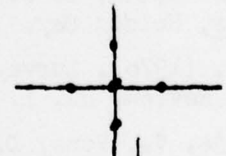
3.2 Other Models. Yule-Walker equations, autocorrelation functions and some other results are obtained for other models m = 2, but are not reported in this paper. Some of these models are:

A. AR (1; 1,1,1,1) Nine point model (10 parameters)

B. Special case of A 5 point model (6 parameters)

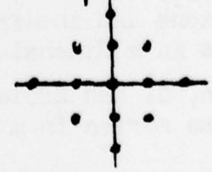
C. With B - AR (2;1,1; 1,1)

5 point model - (11 parameters)



D. Special case of AR (1;,2,2; 2,2)

13 point model (14 parameters)



E. For model given in (3.1.1)

4. Conclusions. The general autoregressive models in m dimensions have been defined and general theorems obtained. The well-known zero dimensional theory is a sub-case. Important cases for m = 1 and 2 have been considered and their properties investigated.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Spatially dependent autoregressive models in m dimensions are defined. The conditions for stationarity and invertibility are determined. The autocorrelation function and Yule-Walker equations are obtained for the general case, and as particular cases $z(t) = f(x_1, x_2; t)$, for $-\infty \le t \le t_0$, for special discrete values x_{1i}, x_{2i}, and t_i, for various grids in (x_1, x_2) plane and for orders 1 and 2 in time. The spectra are obtained for these particular cases,

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and some results for the partial autocorrelation function. All results are new. The notation, definitions and assumptions are those given by Voss et al. (1977). We assume stationarity of $z_{x,t}$ over time t, where $x = (x_1, x_2, \ldots, x_m)$ as m dimensional vector. We assume the covariance structure as given by Hannan (1970), with $\sigma_z^2 > 0$, and all covariances existing. Non-stationarity models will be considered in later papers.

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